Limit of Centrifugal Separation in a Free Vortex

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Two vortex experiments are cited that show that the common analysis of centrifugal separators does not adequately predict the separation of small particles in jet-driven swirl separators. The common analysis ignores the interference of the particle with the vortex flow in cases where the flow deviates from the solid vortex type. This interference, in connection with viscous effects, causes substantial fluid pressure forces opposing the separation of the particle. With the viscous effects approximated by analogy with straight flow conditions, the limit for the separation of particles in a free vortex is derived. It becomes apparent that particles with a density much higher than that of the flow medium can move toward the vortex center instead of being separated. In zones where the flow deviates from the free vortex conditions, the inward movement may be stopped resulting in the stabilization of the particles. The present considerations pertain quite generally to the movement of small particles in curved flows.

Nomenclature

k = proportionality factor
r = particle radius
R = particle orbit radius
Re_t = tangential vortex Reynolds number uR/ν
u = fluid tangential velocity in the free vortex
v = tangential velocity of certain positions on the particle
Δv = velocity difference between particle and fluid
V = particle volume
μ = micron = 10^{-6} m
ν = kinematic viscosity of the vortex fluid at the position of the particle in the vortex
π = 3.14
ρ = density

Superscript

$*$ = "equivalent particle"

Subscripts

c = center of particle
F = vortex fluid
P = particle
1,2 = positions on the particle (see Fig. 1)
s = particle with slowed down rotation

I. Introduction

IN the common separator analysis, the buoyancy forces acting on the particle in the rotating fluid are accounted for by determining the centrifugal forces with a particle mass that is reduced by the mass of the displaced fluid. Under this condition, the limit of separation is obviously reached if particle density and fluid density are the same. In case of density differences where the particle moves aganist the fluid, fluid dynamic drag is the only force opposing the movement of the particle. Although this approach is inherently correct for solid vortex type separators, such as ultracentrifuges,[1] it leads, as this analysis attempts to show, to considerable errors in case of jet-driven swirl separators that exhibit an essentially free vortex type flow. The initiative for the present investigation comes from the following experimental experience:

1) In an intense jet-driven air-swirl condensation droplets of 0.26 μ diam (u = 318 m/sec, R = 0.00645 m; pressure and temperature at particle position: p = 2.04 kg/cm^2, T = 261°K) have an apparent separation velocity which is nearly an order of magnitude smaller than the one calculated in the conventional way with Stokes' drag law (with a Cunningham correction included). The experiment is described in detail in Ref. 2.

2) In a slow jet-driven laminar water swirl particles of nominal 30 μ diameter with about 2.5 times the density of water (u = 0.3 m/sec, R = 0.003 m) become radially stabilized, i.e., they are not separated at all. The experiment described in detail in Ref. 3 also provided evidence for the well-known fact that the radial inflow is restricted to the lateral boundaries of the vortex, and that the bulk of the vortex is essentially free of a radial inflow.

The existence of such large discrepancies between calculated and experimental values for the separation velocities in jet-driven swirls is apparently not known, since no record of similar observations by other investigators could be found. It is however, known that swirl separator performances deteriorate greatly with smaller size particles, say below 5 μ diameter. Secondary effects such as turbulence and re-entrainment are commonly blamed for this lack in performance. Deviations from Stokes' law in rotating flows,[4] rotation of the particles,[5,6] unsteady movements of the particles,[7] and also flow turbulence[8] cannot account for the magnitude of the observed discrepancies. These effects also fail to explain certain peculiarities in the observed behavior of the particles.

For the free vortex an approximate formulation is given in the following that suggests that the discrepancies must be traced to the local interference of the particle with the flow in cases where the flow deviates from the solid vortex type. This is given in the hope to give a plausible explanation of the phenomenon.

II. General Considerations

When the vortex flow changes from a solid vortex (vortex with solid body rotation) toward a free vortex (vortex with constant angular momentum), three effects appear which are well understood but neglected as small in the common analysis of centrifugal separation: 1) The radial buoyancy of the particle, which tends to drive the particle toward the vortex center, increases, 2) the particle obtains an excess rotation against the vortex flow, and 3) due to the difference in rotation a viscous interaction between the particle and the fluid takes place.

The present analysis contends that it depends entirely on the viscous conditions around the particle whether these effects can actually be neglected. These three effects are

Received June 6, 1969; revision received October 20, 1970.
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interrelated and with increasing intensity of the viscous interaction sufficient excess rotation can be produced in the fluid surrounding the particle to induce fluid buoyancy forces that tend to drive the particle toward the vortex center. The following general consideration should make it obvious that small particles are particularly subject to the influence of these effects. The viscous forces acting on a particle are connected with the surface of the particle, while the inertial forces which effect the separation are related to the volume of the particle. With decreasing particle size, i.e., increasing surface to volume ratio of the particle, the viscous influences increase greatly against the inertial ones even in low-viscosity fluids like air or water, a trend well known from Stokes' drag coefficient for a sphere. It is apparent that it depends only on the smallness of the particle how severely the particle interference effect influences the particle separation process.

In the following sections a quantitative analysis of the interference effect, which should allow comparisons with the experiments, is attempted.

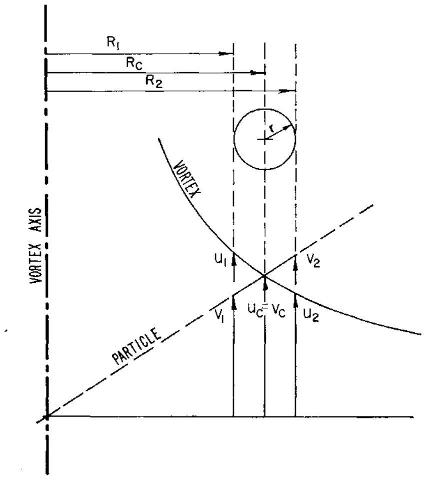

Fig. 1 Radial distribution of the tangential velocities in the free vortex and on the particle rotating around the vortex axis as part of a solid vortex.

III. Radial Equilibrium of the Particle in the Vortex

A particle in a solid vortex is in radial equilibrium, i.e., its separation velocity is zero, if its density is the same as that of the surrounding fluid. This condition, besides being obvious, can be simply derived from equating the radial fluid pressure forces acting on the particle with the centrifugal forces acting on the particle. If we apply such force balance to a particle in a free vortex, we find that the density of the particle must be higher than that of the fluid to obtain radial equilibrium. This is shown here for a cubic particle that allows very simple derivations. Its geometric center should travel around the vortex axis with the same tangential velocity as the fluid as shown in Fig. 1 for a spherical particle. With the cubicle being square to the vortex axis and its side length being $2r$, all notations in Fig. 1 apply also to the case of the cubicle. From potential flow considerations, for the force balance follows:

$$(2r)^2(\rho_F \cdot u_1^2/2 - \rho_F \cdot u_2^2/2) = (2r)^3 \cdot \rho_P \cdot v_c^2/R_c \qquad (1)$$

For the free vortex $u_1/u_c = R_c/R_1$ and $u_2/u_c = R_c/R_2$. From Fig. 1, $u_c = v_c$; $R_2 = R_c + r$ and $R_1 = R_c - r$. Introducing these relations into Eq. (1) the following equation is obtained (writing R_c without index):

$$\rho_F/\rho_P = [1 - (r/R)^2]^2 \qquad (2)$$

This relation shows that the density of the particle must always be higher than that of the fluid to obtain radial equilibrium. Since Eq. (2) cannot depend to any large degree on the particular shape of the particle, we apply this equation also to the spherical particle which we use in the following as more convenient for our further considerations.

IV. Rotation and Position of the Particle

We may assume for a moment that a spherical particle rotates as shown in Fig. 1 around the vortex axis as part of a solid vortex, i.e. its rotational speed around its own axis is the same as that around the vortex axis. Under this condition the differences in tangential velocity between particle and fluid produce a shear force moment that tends to slow down the rotation of the particle around its own axis to adapt it so to speak to the zero rotation condition of the free vortex. If we allow the particle to slow down its rotation, velocity differences in radial direction appear on the particle and with them a shear force pair which opposes the slowdown of its rotation. It is now obvious, and it can also be shown experimentally, that the particle rotates around its own axis with about half the speed with which it rotates around the vortex axis to bring the shear force moments into equilibrium. Because of the hyperbolic dependence of the fluid tangential velocity in the vortex, the velocity difference and correspondingly the shear forces are on the inner side of the particle at R_1 slightly higher than on the outer side at R_2. Forming the moment of these forces with their respective distances from the vortex center, the sum of these moments tends toward zero indicating that the movement of the particle around the vortex axis is not affected by the shear forces acting on the particle.

Fluid shear forces are inherently connected with kinetic energy dissipation. This energy loss may be qualitatively accounted for as follows. The particle is driven by the fluid on the inside at R_1, while the particle itself drives the fluid at R_2. A measure for the kinetic energy transferred from the fluid to the particle and given off by the particle on the fluid is the difference of the square of the absolute velocities involved, $(u_1^2 - v_{1-s}^2)$ and $(v_{2-s}^2 - u_2^2)$ respectively, with v_{1-s} and v_{2-s} being the absolute tangential velocities of the particle at R_1 and R_2, respectively, in its state of reduced rotation. Since the average absolute velocities involved are higher at R_1 than at R_2, the difference of the squared values is also higher at the inside at R_1 than on the outside at R_2. This means more energy is given off by the fluid at R_1 than received by the fluid at R_2. The difference covers the losses in this energy transfer by shear forces.

From these momentum and energy considerations it appears reasonable to assume for our analysis a particle the geometric center of which moves with the fluid as indicated in Fig. 1 with $u_c = v_c$, and rotates around its own axis with only half the speed with which it rotates around the vortex axis. This makes the tangential particle velocity at R_1 and R_2 very nearly equal and the tangential velocity differences between the particle and the fluid only half as large as shown in Fig. 1.

V. Equivalent Particle

Oseen's solution for the flow around a sphere moving through a viscous fluid under straight flow conditions suggests a very helpful simplification of the present flow problem. In this solution the flowfield is represented in its essence by a wake flow directed toward the sphere, changing around the sphere to a source flow.[9] There is no reason why this basic flow scheme should not in principle be applicable to curved flow conditions and in particular to our particle in the vortex.

In assuming Oseen's basic flow picture to be valid in the present case, our flow problem is greatly simplified for the following reason. Only the source flow determines in its essence the pressure field induced by the particle in the fluid.

Thus, the fluid pressure forces that are important for the radial force balance on the particle are only a function of potential flow conditions. The fact that there is no sink flow is consistent with the existence of a drag on the particle, i.e., the pressure forces induced by the source flow in the fluid are balanced by forces originating from the particle. If we would replace these forces by the fluid pressure forces induced by a sink flow, the fluid pressure distribution around the particle would not be drastically changed particularly not in the radial vortex direction which is important for the radial equilibrium of the particle. Thus the flowfield essential for the particle equilibrium can be represented by purely potential flow elements. We can replace the particle by a sphere under nonviscous conditions where the size of the sphere is determined by the requirement that the source flow produced by the sphere is the same as that produced by the particle in the real fluid. Eq. (2) can be directly applied to this "equivalent particle" if we also assume that the density of the equivalent particle is the average density of a sphere which consists of the actual particle and vortex fluid surrounding it.

This latter assumption implies a rigid coupling between particle and fluid. Actually the coupling involves a fluid-dynamic process. The fluid pressure field induced by the particle causes fluid around the particle to flow toward the vortex center. Viscous forces generated by this inflow on the particle counteract the centrifugal forces which tend to separate the particle. The pressure field due to this new interaction is as the one of the primary interaction spread over a comparatively wide area in the fluid giving the viscous coupling process a high degree of rigidity. Since Coriolis forces tend to impede the induced flow movement, they intensify the viscous interaction process and add to the rigidity of the coupling action.

For equal source flow on the equivalent and the real particle, the dynamic pressure forces acting on the equivalent particle must equal the viscous drag on the real particle. Difficulties arise in determining the particle drag under vortex conditions where the freestream velocity increases with the distance from the particle. It is, however, reasonable to assume that Stokes' drag law still formally applies in this case, i.e., that a characteristic velocity difference between particle and fluid can be defined that gives with Stokes' law the correct drag. The fact that the particle in the vortex is exposed to two opposing flows as evident from Fig. 1 is of little consequence here since the source flow is independent of the flow direction, at least in the far field around the particle. We can therefore equate the viscous drag with the dynamic pressure forces in a formal way as follows:

$$6\pi\cdot\nu\cdot\rho_F\cdot\Delta v\cdot r = (r^*)^2\cdot\pi\cdot\rho_F\cdot(\Delta v)^2/2 \tag{3}$$

From this relation for the radius of the equivalent particle follows:

$$r^* = [(12\cdot\nu\cdot r)/(\Delta v)]^{1/2} \tag{4}$$

For the characteristic velocity difference Δv, it can be assumed that it is in a certain proportion to the velocity difference $(u_1 - v_{1-s})$ on the particle at R_1 (see Sec. IV). Using the tangential velocity gradient du/dR of the vortex flow the proportionality of Δv can be expressed in the following way:

$$\Delta v = (du/dR)\cdot r\cdot k \tag{5}$$

There is no simple way to calculate the proportionality factor k in this equation. However, a new water experiment has been carried out which allows us to introduce an empirical value for k. In this experiment, described in more detail in Sec. VII, true particle equilibrium was established. (It will be seen in Sec. VII that in the original water experiment, cited in Sec. I, the particles were not in radial equilibrium in the sense of the present considerations.) To satisfy the analysis in applying it to this new water experiment the factor k must have a value of 3. This value appears very reasonable. It indicates that the near-field conditions around the particle determine, as expected, the particle drag.

Considering that for the free vortex $du/dR = u/R$, Eq. (5) can be written

$$\Delta v = (u/R)\cdot r\cdot k \tag{6}$$

Inserting this equation into Eq. (4) we obtain

$$r^* = [(12\cdot\nu\cdot R)/(u\cdot k)]^{1/2} \tag{7}$$

Surprisingly, the original particle radius cancels out. If we refer r^* to the distance R of the particle from the vortex axis and introduce the tangential Reynolds-number of the vortex $Re_t = u\cdot R/\nu$, we obtain the following simple relation for the dimensionless radius of the equivalent particle:

$$r^*/R = [12/(k\cdot Re_t)]^{1/2} \tag{8}$$

There are certain effects which have not been considered in the analysis except indirectly by the empirical factor k. In rotating flows Coriolis forces increase the viscous drag.[4] We have also neglected the wake flow for its contribution to the particle disturbance. The wake flow may become important, if the particle has a separation velocity, since a radial component of the wake appears which is liable to induce radial pressure forces impeding the radial particle movement.

VI. Minimum Density Ratio for Separation

Equation (2) can be used now to determine the radial equilibrium conditions of the equivalent particle. We define in this equation r as the equivalent radius r^* and ρ_P as the density $\rho_P{}^*$ of the equivalent particle. Together with Eq. (8) we find:

$$\rho_F/\rho_P{}^* = [1 - 12/(k\cdot Re_t)]^2 \tag{9}$$

Considering that the equivalent particle is made up of the actual particle and surrounding fluid, its average density is

$$\rho_P{}^* = [\rho_P\cdot V_P + \rho_F(V_P{}^* - V_P)]/V_P{}^* \tag{10}$$

With some simple transformations Eq. (10) becomes

$$\rho_P{}^*/\rho_F = 1 + (r/R)^3\cdot(R/r^*)^3\cdot[(\rho_P/\rho_F) - 1] \tag{11}$$

Introducing this relation into Eq. (9), we arrive at the final result (designating the equilibrium density ratio as the minimum density ratio for separation)

$$\left(\frac{\rho_P}{\rho_F}\right)_{\min} - 1 = \left(\frac{R}{r}\right)^3\cdot\left(\frac{12}{k\cdot Re_t}\right)^{5/2}\cdot\left[\frac{2 - 12/(k\cdot Re_t)}{[1 - 12/(k\cdot Re_t)]^2}\right] \tag{12}$$

In our present considerations, we are interested only in Re_t-values above about 10^3. For these values the last term in Eq. (12) in brackets is very close to 2. If we also insert for k in this equation the value 3 as discussed in Sec. V., we arrive at the equation for our numerical evaluations

$$(\rho_P/\rho_F)_{\min} - 1 = 70\cdot(R/r)^3\cdot(Re_t)^{-5/2} \tag{13}$$

This relation is represented in Fig. 2 by the straight lines marked by the parameter Re_t. The application of this plot to the experiments is discussed in the next section.

VII. Separation Limits in the Swirl Experiments

The minimum density ratios required for separation in the experiments cited in Sec. I are indicated in Fig. 2 (open circles). For the free vortex regime of the water swirl, the parameter Re_t does not change with the radial position R of the particle and is therefore constant over r/R. For the air swirl Re_t changes with R due to the change of the air density over the radius of the vortex. In this case two r/R ranges are shown corresponding to the two zones which have been observed in the condensate particle cloud of this experiment: an inner zone containing predominantly particles of 0.26μ and an adjacent outer zone with 0.31μ particles (the center region of the vortex is free of droplets; there are also no droplets outside the outer zone). The actual particle to fluid density ratios for both experiments have also been marked (solid circles) in Fig. 2 for comparison with the minimum density ratios.

Air experiment

We see that for the air swirl depending on the specific conditions the particle density must be from about 200 to 600 times higher than the air density at vortex conditions to accomplish separation of the particle. The minimum density ratios cross over the actual density ratios in the outer zone for the 0.31μ diameter particles, i.e., at the crossover point these particles are in radial equilibrium. In general it becomes apparent that the particles are very close to radial equilibrium in spite of the fact that their density is by more than two orders of magnitude higher than that of the vortex fluid. This finding is in general agreement with the cited experimental experience. In the following, it will be shown that this result allows also a very consistent explanation of the particle behavior in the air experiment.

For a discussion of the air experiment the following influences must be considered: 1) The particles, which must be assumed to originate in the inner zone by spontaneous condensation, are small enough to be subject to slip flow conditions, reducing the minimum density ratio by a factor of about 1.5 to 2. 2) The air swirl has a radial inflow. In this experiment the inflow through the lateral boundaries was purposely suppressed by a special air injection system. Thus, the through flow must have taken place, at least in part, through the vortex itself. (A radial inflow uniform over the length of the vortex constitutes a super-position of two potential flow systems and is therefore compatible with free vortex conditions.) 3) The inner zone with the 0.26μ particles also contains the axial discharge flow of the vortex extending practically over the whole length of the vortex. This axial flow causes fluid shear and therefore deviations from free vortex conditions in this zone. 4) Extremely steep tangential velocity gradients exist in the inner vortex region. They are only explainable if the turbulence level in this region is very low. However, macroscopic flow fluctuations (not necessarily associated with turbulence) have been observed in the inner region of the vortex.[10] 5) There is practically no particle size variation over the radial extent of the outer zone containing the 0.31μ particles.

Influences 1 and 2 are opposing and nearly cancel each other. On the basis of our analytical results and the aforementioned influences we may attempt to explain the air experiment in the following way: The particles originate in the inner zone and do not separate as long as they are much smaller than 0.26μ. Since there is in this zone the axial discharge flow, they will move toward the swirl chamber outlet. They stay, however, out of the central solid vortex type region of the vortex. While the droplets grow by further condensation, a part of them will leave the swirl chamber with the axial flow. Others close to the outer zone will grow to near 0.26μ size and move to the outer zone. The deviation from free vortex condition in the inner zone will favor this separation movement. While still growing in a transition zone to 0.31μ size, the droplets will enter the outer zone and obtain more or less radial equilibrium. With the supersaturation in the vortex environment decreasing in this zone, the growth rate for the droplets will diminish and the particles obtain an over-all stabilized state. Since the equivalent radius of the particles as given by Eq. (8) exceeds in this zone the average distance between the particles, which is about 100 particle

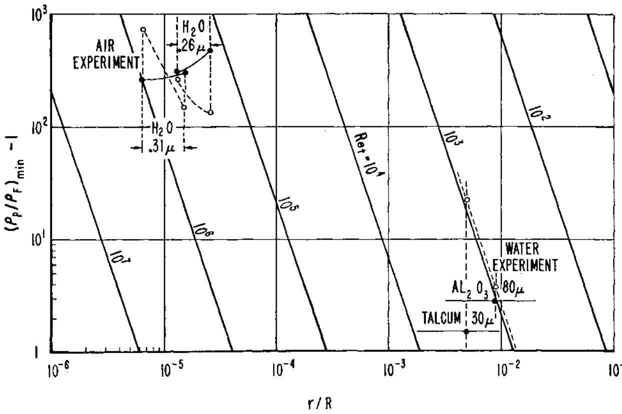

Fig. 2 Minimum particle to fluid density ratio ρ_P/ρ_F necessary to achieve separation in a free vortex, plotted in the form $[(\rho_P/\rho_F) - 1]$ over the particle to particle orbit radius r/R with the tangential vortex Reynolds number Re_t as parameter. Points marked by open circles denote the minimum density ratio for separation applicable to the indicated experiments. The ordinate values are also used to mark (solid circles) the actual particle to fluid density ratios which existed in these experiments.

diameters, mutual viscous interference between the particles (avalanche effect) may again promote separation and allow the particles to reach the outer unsaturated regions of the chamber where they will evaporate.

The present analysis allows a reasonable explanation of the particle behavior in the air experiment without reference to turbulence effects. It removes the claim which conventional explanations need that the turbulence level in the air swirl is high, say in the order of 1%. Direct evidence that the turbulence level in the inner vortex region is actually very low is shown by the existence of individual particle zones and very steep velocity gradients there. Reference 8 shows also that a free vortex has a tendency to suppress turbulence. There is also the effect of sound waves being scattered by a free vortex as, for instance, dealt with in Ref. 11. This effect would also indicate that flow disturbances do not readily penetrate into a free vortex. From these various effects, which find their basic explanation in the well known tendency of rotating flows to establish two-dimensional conditions (pertinent literature cited in Ref. 3), it appears that a very low-turbulence level is inherent to free vortex type swirls.

Water experiment

As already mentioned in Sec. V, experimental observations have shown that the "80μ alumina" particles are in radial equilibrium in the free vortex portion of the water vortex (see description below). This condition was used to determine the magnitude of the proportionality factor k in Eq. (5). The uniqueness of the value 3 found for k becomes apparent from the application of the analysis to the other experiments. Though the air experiment differs greatly from the water experiment in the vortex and particle conditions as evident from Fig. 2, the analysis is in reasonable agreement with both experiments. For the air experiment this has been shown in the preceding chapter. For the original water experiment with the "30μ talcum" particles it will be shown in the following that the analysis is also consistent with this experiment and that it is particularly revealing for the behavior of the particles in this experiment.

As Fig. 2 shows, the minimum density ratio for these particles is appreciably above the actual one, i.e., these particles must move toward the vortex center. The analysis now allows an intriguing interpretation of the apparent stabilization observed with these particles in the experiment.

If the particles move toward the vortex center, they can do this only as long as the flow is sufficiently near free vortex conditions. Since the flow changes to near solid vortex condition in the center of the vortex, the particles are stopped in the transition region. The strong axial flow in the inner core of the vortex gives this region a very smooth axial contour. It is also the cause for the deviation of this region from free vortex conditions. This axial flow is not compatible with potential flow conditions, as already indicated in the air experiment. The smooth inner core which stops the particles in their radial movement also gives them, while aligning along this core, an extremely smooth shape, as can be seen in the photograph of Ref. 3. The particles in this experiment exhibit almost zero sinking speed. One must conclude that a weak upwardly directed axial flow surrounds the center flow canceling the sinking speed of the particles. This axial shear flow layer acts as a barrier to the particles preventing them from being swallowed by the inner discharge flow.

To check whether the transition zone between the free vortex and the inner core region can trap particles of different sizes as the analysis would indicate, the water experiment was repeated and particles contained in the stabilized formation were recovered and checked for size. For the talcum particles it was found that the recovered particles range from 30μ to 60μ and oblong particles up to 100μ were found. Particles smaller than 30μ apparently escaped to the vortex center.

In the ensuing attempt to find the separation limit, larger particles consisting of alumina (Al_2O_3, $\rho_P = 3.8$ g/cm^3) were added to the vortex. With particles of about 50μ–80μ diam, the separation limit was apparently being approached. The particle formation was no longer very smooth and particles changed to distinctly larger orbits. The condition for the 80μ particle at 9 mm orbit radius is indicated in Fig. 2.

VIII. Conclusion

The present analysis shows that viscous effects can decisively influence the radial force equilibrium for a particle in a free vortex (or in any rotating flow deviating sufficiently from a solid vortex structure). In spite of the use of greatly simplifying assumptions the analysis allows a meaningful comparison with experiments and gives an improved insight into the particle behavior in a free vortex type swirl. Though further experimental checks on the analysis are desirable for a better assessment of the accuracy of the analysis, the present results already demonstrate that the conventional method of calculating the performance of free vortex type separators (or particle trajectories in curved flows in general) needs some revision.

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